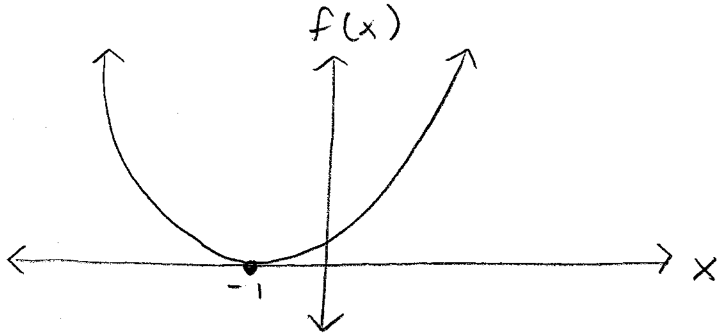
MA 131 HW # 5

Section 2.2

2.) b, c, f

4.) f

8.)



19.)

	f	f'	f''
A	+	+	−
B	0	−	0
C	−	0	+

21.) Faster at $t=1$ since derivative is larger there

22.) Faster at $t=2$. Graph is of velocity and y-value bigger at $t=2$

23. a) Decreasing

b.) Rel max since $f'(x)$ changes from $+$ to $-$. Rel max $= (2, 9)$

c) $f'(x)$ changes from $-$ to $+$

d) Concave down

e) Inflection at $(6,5)$

f) $x = 15$ since $f'(15) = 6$

43. a) $f(65) \approx 2$ million farms

b) $f'(65) \approx -0.025$ million forms / year

c) $f(15) \approx 6$ million. $\boxed{1940}$

d) $60{,}000 = .06$ million

$f'(20) \approx -.06$

$f'(54) \approx -.06$

In 1945 and 1979

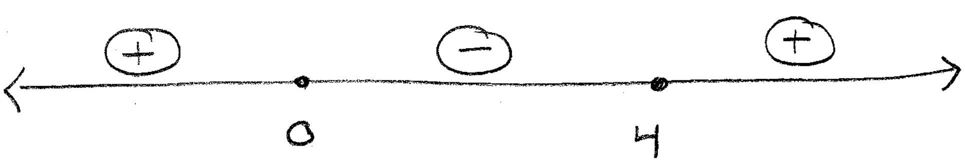
e) 1960

Section 2.3

2.) $f'(x) = 3x^2 - 12x$

$0 = 3x^2 - 12x = 3x(x-4)$

$x = 0$ OR $x = 4$



$f'(-1) = 3 + 12 = 15$

$f'(1) = 3 - 12 = -9$

$f'(5) = 3 \cdot 25 - 12 \cdot 5 = 75 - 60 = 15$

$f(0) = 1$

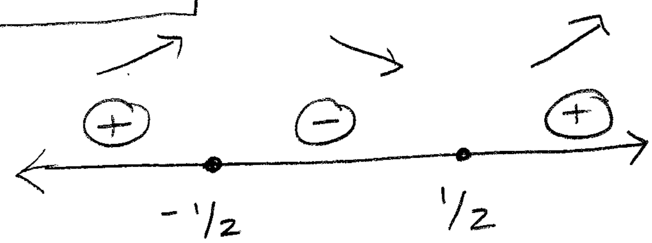
$\boxed{\text{Relative max at } (0,1)}$

$f(4) = 64 - 6 \cdot 16 + 1$

$= 64 - 96 + 1 = -31$

$\boxed{\text{Relative min at } (4, -31)}$

6.) $f'(x) = 4x^2 - 1$



$0 = 4x^2 - 1$

$1 = 4x^2$

$\frac{1}{4} = x^2$

$\pm \frac{1}{2} = x$

$f'(-1) = 3$

$f'(0) = -1$

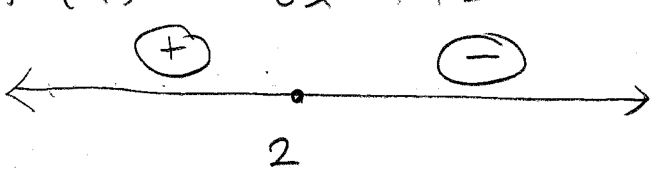
$f'(1) = 3$

$f(-1/2) = \frac{4}{3}(-1/2)^3 + 1/2 + 2 = 7/3$

$f(1/2) = \frac{4}{3}(1/2)^3 - 1/2 + 2 = 5/3$

$\boxed{\begin{array}{l} \text{Rel Max: } (-1/2, 7/3) \\ \text{Rel Min: } (1/2, 5/3 \end{array}}$

12.) $f'(x) = -6x + 12$



$0 = -6x + 12$

$6x = 12$

$x = 2$

$f'(0) = 12$

$f'(3) = -18 + 12 = -6$

Increasing: $x < 2$

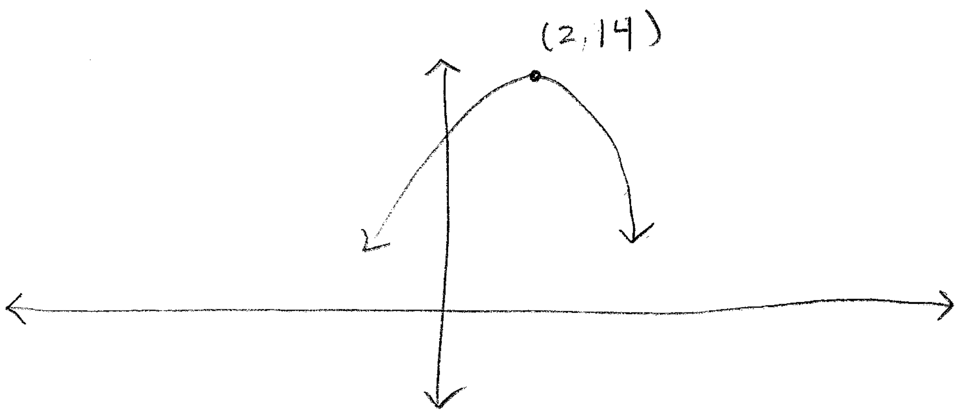
Decreasing: $x > 2$

$f(2) = -3 \cdot 4 + 12 \cdot 2 + 2$

$= -12 + 24 + 2 = 14$

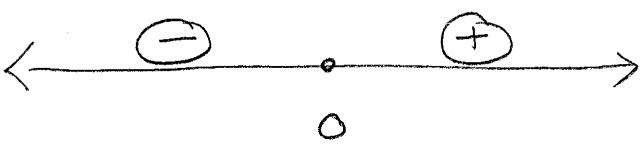
Rel max at $(2, 14)$

$f''(x) = -6$ so concave down everywhere



14.) $f'(x) = x$

$0 = x$



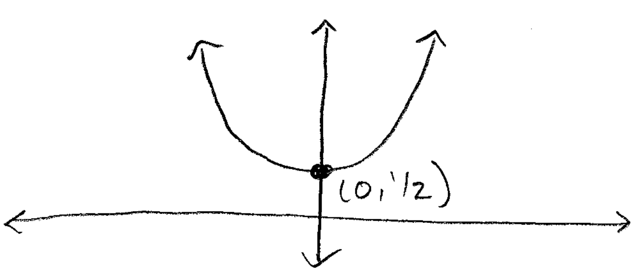
Increasing for $x > 0$

Decreasing for $x < 0$

$f(0) = 1/2$

Rel min at $(0, 1/2)$

$f''(x) = 1$ so concave up everywhere

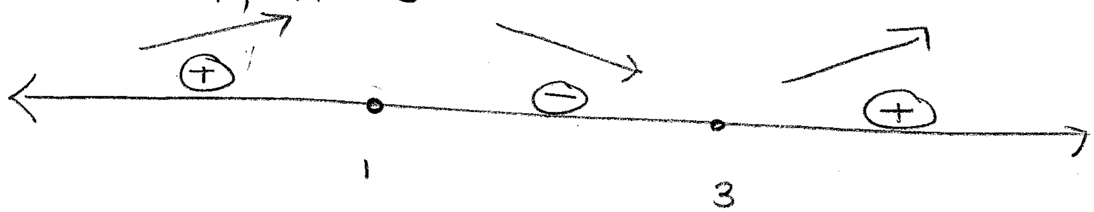


26.) $0 = y' = 3x^2 - 12x + 9$

$= 3(x^2 - 4x + 3)$

$= 3(x - 3)(x - 1)$

$x = 1, \quad x = 3$



$f'(0) = 3(-3)(-1) > 0$

$f'(2) = 3(-1)(1) < 0$

$f'(4) = 3(1)(3) > 0$

$f(1) = 1 - 6 + 9 + 3 = 7$

$f(3) = 27 - 6 \cdot 9 + 27 + 3 = 54 - 54 + 3 = 3$

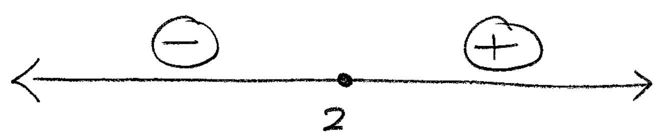
$\boxed{\begin{array}{l} \text{Rel max: } (1, 7) \\ \text{Rel min: } (3, 3) \end{array}}$

$f''(x) = 6x - 12$

$0 = 6x - 12$

$12 = 6x$

$2 = x$



$f''(3) = 6 > 0$

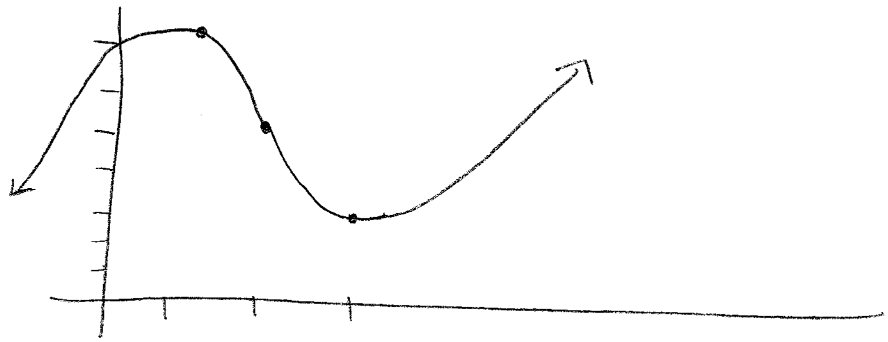
$f''(0) = -12 < 0$

Concave Up: $x > 2$

Concave Down: $x < 2$

$f(2) = 8 - 6 \cdot 4 + 9 \cdot 2 + 3 = 8 - 24 + 18 + 3 = 5$

Inflection Point: $(2, 5)$

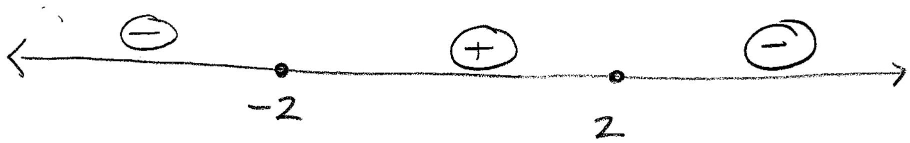


28.) $y' = -3x^2 + 12 = 0$

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$



$y'(-3) = -3 \cdot 9 + 12 = -27 + 12 < 0$

$y'(0) = 12 > 0$

$y'(3) = -3 \cdot 9 + 12 = -27 + 12 < 0$

Increasing: $(-2, 2)$

Decreasing: $(-\infty, -2) \cup (2, \infty)$

$y(-2) = -(-2)^3 + 12(-2) - 4 = 8 - 24 - 4$

$= -20$

$\boxed{\text{Rel min at } (-2, -20)}$

$y(2) = -8 + 24 - 4 = 12$

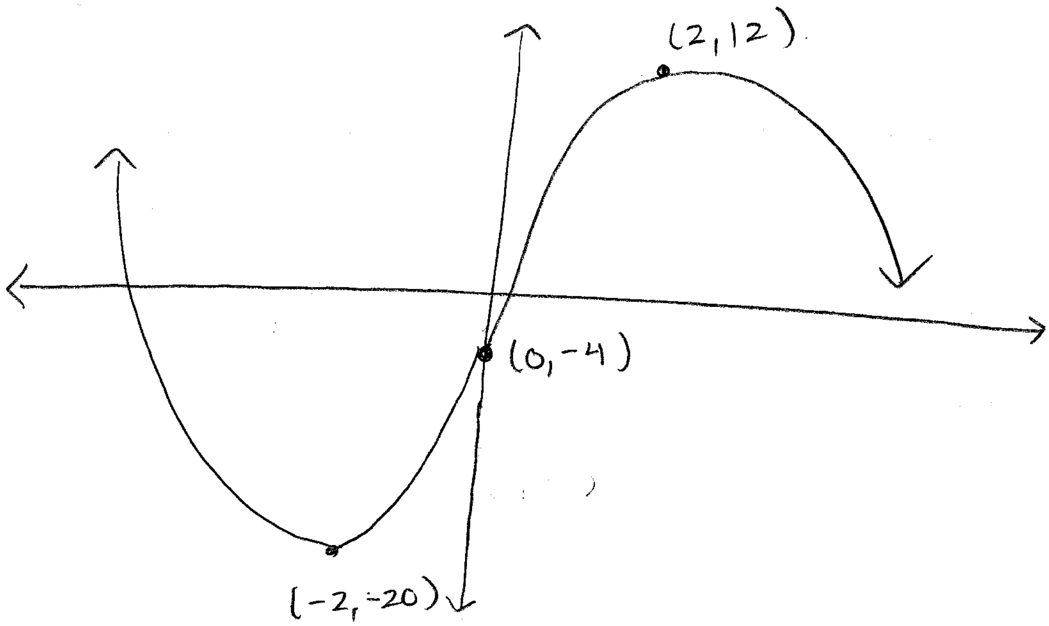
$\boxed{\text{Rel Max at } (2, 12)}$

$y'' = -6x$

Concave Up for $x < 0$

Concave Down for $x > 0$

Inflection Point: $(0, -4)$



41.) $g(x)$ always increasing, so $g'(x) > 0$ for all x. Since $f(x)$ is negative for some x, $f(x)$ is not the derivative of $g(x)$.

$g(x)$ is derivative of $f(x)$

42.) $g(x)$ is decreasing for some x, so $g'(x)$ is negative for these x. Since $f(x)$ is always positive, $f(x)$ not derivative of $g(x)$. $g(x)$ is derivative of $f(x)$

43. a) Relative min at $x = 2$. Deriv changes from $-$ to $+$

b) Inflection point

44. a) $f(125) = 125$ million

b) $f(50) \approx 25$. 1850

c) $f'(150) \approx 2.25$ million people/year

d) $f'(175) = 1.8$. 1975

e) Max growth rate at $t \approx 140$. 1940

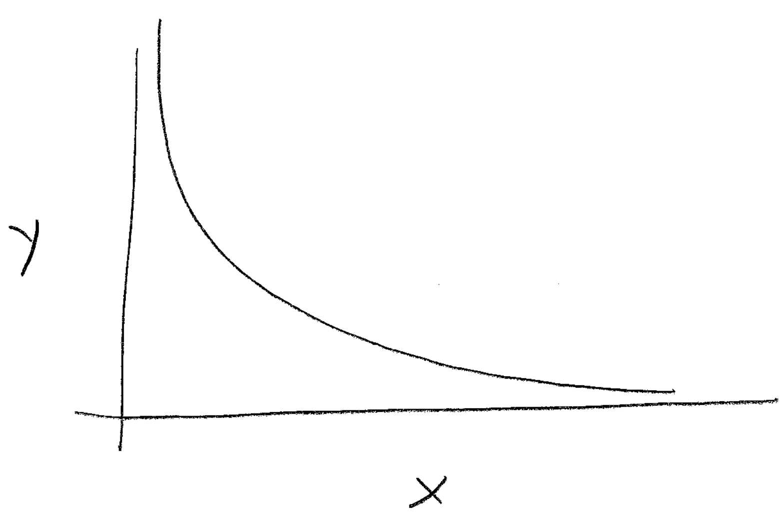
Section 2.4

24.) $y' = -\frac{2}{x^2} < 0 \longrightarrow$ y always decreasing

$y'' = \frac{4}{x^3} > 0 \longrightarrow$ y always concave up

Vertical Asymptote at $x = 0$

$\lim_{x \to \infty} 2/x = 0$



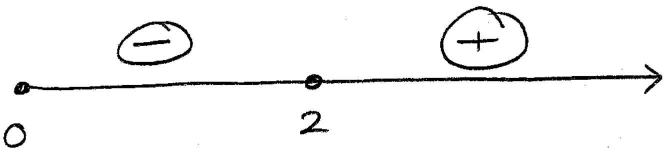
26.) $y' = -\frac{12}{x^2} + 3 = 0$

$\frac{12}{x^2} = 3$

$12 = 3x^2$

$4 = x^2$

$\pm 2 = x$



$y'(1) = -12 + 3 < 0$

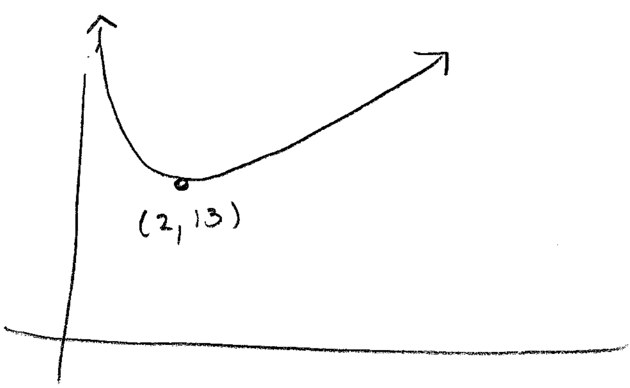
$y'(3) = -\frac{12}{9} + 3 > 0$

$y(2) = \frac{12}{2} + 6 + 1 = 13$

Relative min at (2, 13)

$y'' = \frac{+24}{x^3} > 0$ for $x > 0$

Concave up



3.1.) When $x \approx 0$, $g(x)$ is decreasing but $f(x)$ positive so $f(x)$ not derivative of $g(x)$. $g(x)$ is derivative of $f(x)$

32.) At the far left of graph, $g(x)$ <u>decreasing</u> but $f(x)$ <u>positive</u>, so $f(x)$ not the derivative of $g(x)$. $g(x)$ is the derivative of $f(x)$

36.) $f'(x)$ decreasing at $x=a$ means $f''(a)<0$, and derivative test means local max at $x=a$