MA 131 HW # 6

Optimization Worksheet

1.) Profit = Revenue − Costs

$= np - 1000 - 100n$

$= n\left(1000 - \frac{n}{10}\right) - 1000 - 100n$

$= 1000n - \frac{n^2}{10} - 1000 - 100n$

$$\boxed{\text{Profit} = \frac{-n^2}{10} + 900n - 1000}$$

$n = 10000 - 10p$

$10p = 10000 - n$

$p = 1000 - \frac{n}{10}$

$n \geq 0$, $n \leq 10000$ (amt they would sell if $p=0$)

$0 = \frac{d}{dn}\text{Profit} = -\frac{n}{5} + 900$

$\frac{n}{5} = 900$

$n = 4500$

$\text{Profit}(n=4500) = \frac{-4500^2}{10} + 900 \cdot 4500 - 1000 = 2,024,000$

$\text{Profit}(n=0) = -1000$

$\text{Profit}(n=10000) = \frac{-10000^2}{10} + 900 \cdot 10000 - 1000 = -1,001,000$

Profit maximized by selling 4500 TV's
Maximum Profit = \$2,024,000

2.) $R = np = (200 - 1.5p)p = 200p - 1.5p^2$

We know $0 \leq n \leq 100$

If $n = 0$, $p = 200/1.5 \approx 133.33$

If $n = 100$, $p = 100/1.5 \approx 66.66$

So p is between 66.66 and 133.33

$0 = R' = 200 - 3p$

$3p = 200$

$p = 200/3 \approx 66.66$

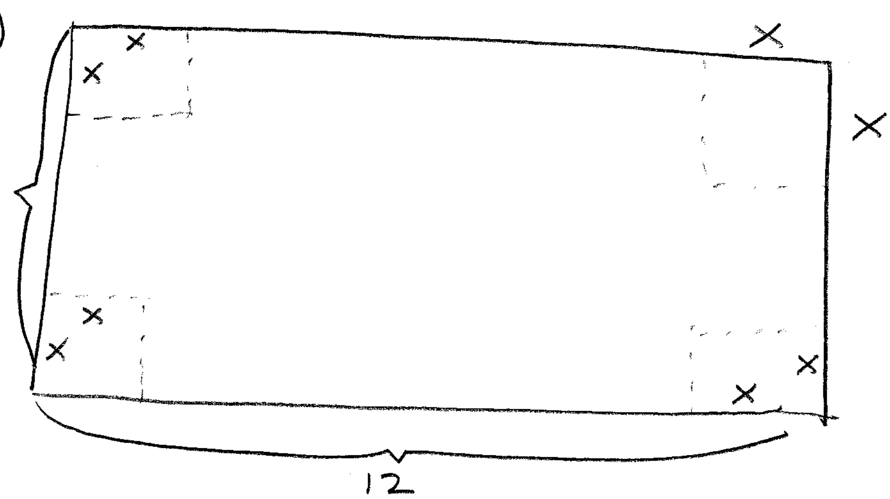
Max Revenue occurs at $p = 66.66$ <u>or</u> 133.33

$R(p = 66.66) \approx \$6666$

$R(p = 133.33) \approx \$0.66$

$$\boxed{\text{Revenue is maximized by setting price to \$66.66/night, at which point } R \approx \$6666}$$

6.)



$V = \text{Length} \times \text{Width} \times \text{Height}$

$= (12-2x)(8-2x)\,x$

$= (96 - 16x - 24x + 4x^2)\,x$

$= (96 - 40x + 4x^2)\,x$

$$\boxed{V = 96x - 40x^2 + 4x^3}$$

$0 = V' = 96 - 80x + 12x^2$

$= 4(24 - 20x + 3x^2)$

$0 = 24 - 20x + 3x^2$

$$x = \frac{20 \pm \sqrt{400 - 4 \cdot 3 \cdot 24}}{6}$$

$x = 1.56$ or $x = 5.09$

We know $x \geq 0$ and $x \leq 4$ (since $2x \leq 8$)

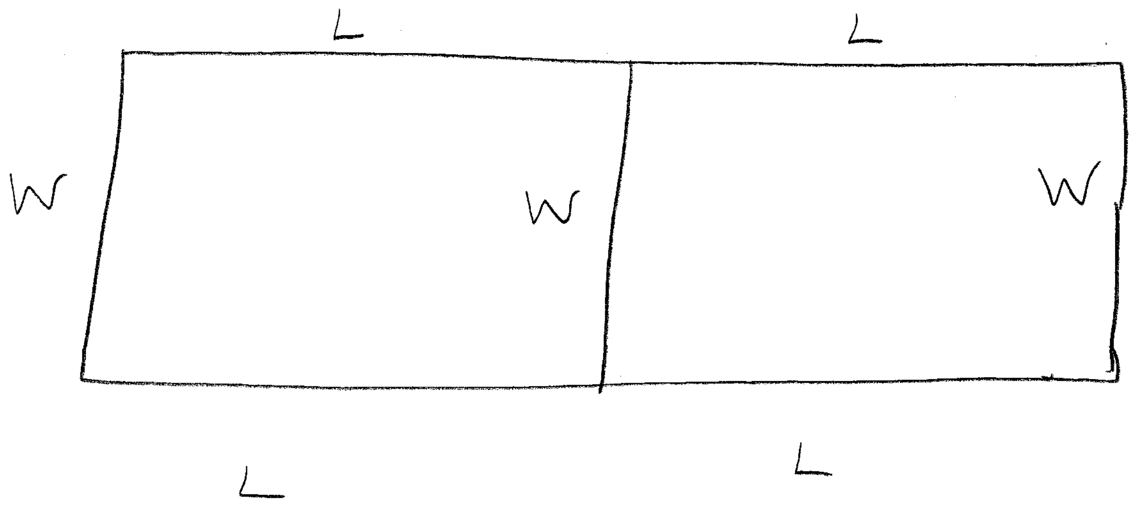
$V(0) = 0$

$V(1.56) = 67.6$

$V(4) = 0$

To get max volume, cut squares of side length 1.56. Max Volume ≈ 67.6

8.) You should arrange the fence like this



$A = LW$ ← Objective Eq

$4L + 3W = 100$ ← Constraint Eq

$3W = 100 - 4L$

$W = \dfrac{100 - 4L}{3}$ ← Plug this into Obj Eq

$$A = L\left(\frac{100-4L}{3}\right) = \frac{100}{3}L - \frac{4}{3}L^2$$

$$0 = A' = \frac{100}{3} - \frac{8}{3}L$$

$$\frac{8L}{3} = \frac{100}{3}$$

$$24L = 300$$

$L = \dfrac{300}{24} = 12.5$ ← Critical Point

$0 \le L \le 25$ (since $4L \le 100$)

$A(L=0)=0$

$A(L=12.5)=\frac{100}{3}12.5-\frac{4}{3}12.5^2\approx 208.33$

$A(L=25)=0$

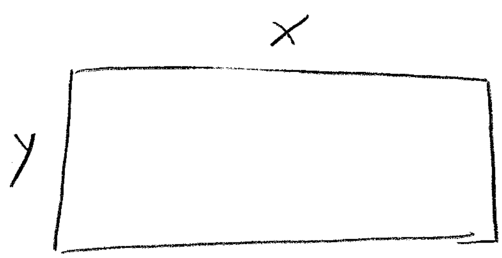
Maximum Area $= 208.33$

This max is attained by setting $L=12.5$,

$$W=\frac{100-4\cdot 12.5}{3}=\frac{50}{3}$$

<u>Section 2.5</u>

11. a)



Objective: $A=xy$

Constraint: $6x+2(x+2y)=320$

b.) $6x+2x+4y=320$

$8x+4y=320$

$4y=320-8x$

$y=80-2x$

$A=x(80-2x)=80x-2x^2$

c) $0=A'=80-4x$

$4x=80$

$x=20$

We know $0 \le x \le \frac{320}{8} = 40$

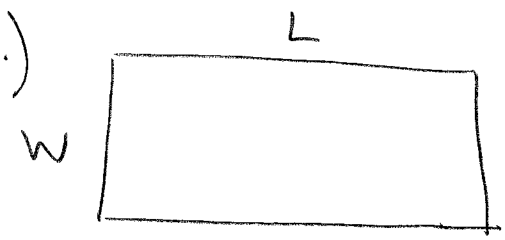
$A(x=0) = 0$

$A(x=20) = 80 \cdot 20 - 2 \cdot 20^2 = 800$

$A(x=40) = 0$ since $y = 80 - 2 \cdot 40 = 0$

$$\boxed{\begin{array}{l} \text{Optimal } x = 20 \\ \text{Optimal } y = 40 \\ \text{Optimal } A = 800 \end{array}}$$

20.)



$A = LW$

$300 = 2L + 2W$

$300 - 2L = 2W$

plug in $150 - L = W$

$A = L(150 - L) = 150L - L^2$

$0 = A' = 150 - 2L$

$2L = 150$

$L = 75$

<u>Bounds:</u> $0 \le L \le 150$

$A(L=0) = 0$

$A(L=75) = 150 \cdot 75 - 75^2 = 5625$

$A(L=150) = 0$

$$\boxed{\begin{array}{ll} \text{Optimal} & L = 75 \\ \text{Optimal} & W = 150 - 75 = 75 \\ \text{Optimal} & A = 5625 \end{array}}$$

<u>Section 2.7</u>

9.) Profits = Revenue − Costs

$= x(256 - 50x) - 182 - 56x$

$= -50x^2 + 256x - 182 - 56x$

$= -50x^2 + 200x - 182$

$0 = \frac{d}{dx}\text{Profits} = -100x + 200$

$100x = 200$

$x = 2$

Bounds: $0 \leq x \leq 5$

Profits $(x=0) = -182$

Profits $(x=2) = -50 \cdot 4 + 200 \cdot 2 - 182$

$= -200 + 400 + 182 = 382$

Profits $(x=5) = -432$

Profit is maximized at 382 by setting $x=2$

Section 3.1

2.) $y' = (-3x^2)\left(\frac{x}{2} - 1\right) + (-x^3 + 2)\left(\frac{1}{2}\right)$

$= -\frac{3}{2}x^3 + 3x^2 - \frac{x^3}{2} + 1$

$= -2x^3 + 3x^2 + 1$

3.) $y' = (8x^3 - 1)(-x^5 + 1) + (2x^4 - x + 1)(-5x^4)$

$= -8x^8 + x^5 + 8x^3 - 1 - 10x^8 + 5x^5 - 5x^4$

$= -18x^8 + 6x^5 - 5x^4 + 8x^3 - 1$

8.) $y' = 4\left[(-2x^3 + x)(6x - 3)\right]^3 \frac{d}{dx}\left[(-2x^3 + x)(6x - 3)\right]$

$= 4\left[(-2x^3 + x)(6x - 3)\right]^3 \Big((-6x^2 + 1)(6x - 3)$

$+ (-2x^3 + x)(6) \Big)$

11.) $y' = \dfrac{x + 1 - (x - 1)}{(x+1)^2} = \dfrac{2}{(x+1)^2}$

12.) $y = (x^2 + x + 7)^{-1}$

$y' = (-1)(x^2 + x + 7)^{-2}(2x+1) = -\dfrac{2x+1}{(x^2 + x + 7)^2}$

13.) $$y' = \frac{(x^2+1)(2x) - (x^2-1)(2x)}{(x^2+1)^2}$$

$$= \frac{\cancel{2x^3} + 2x - \cancel{2x^3} + 2x}{(x^2+1)^2}$$

$$= \frac{4x}{(x^2+1)^2}$$

18.) $$y' = \frac{(cx+d)a - (ax+b)c}{(cx+d)^2} = \frac{ad - bc}{(cx+d)^2}$$

20.) $$y' = \frac{(x^2+1)^2 2x - x^2 \cdot 2(x^2+1)2x}{(x^2+1)^4}$$