Name: Answer Key

MA 131 Test 2 Form B

1. In the space below, draw a graph of the following function:

$$f(x) = \frac{x^3}{3} - 25x + 472\pi^7$$

To help you draw your graph, answer the following questions:

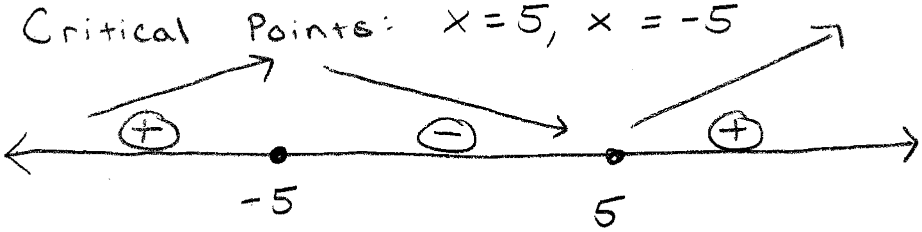
1. Where are the relative maxima and relative minima?
2. Where is $f(x)$ increasing and decreasing?
3. Where is $f(x)$ concave up/concave down?

Do NOT worry about what the exact y-values are for your graph. On your graph, clearly mark the x-values of your critical points and draw a curve that is increasing/decreasing and concave up/down in the right places.

$f'(x) = x^2 - 25$

$0 = x^2 - 25 = (x-5)(x+5)$

Critical Points: $x = 5$, $x = -5$



$f'(-6) = (-11)(-1) > 0$

$f'(0) = -25 < 0$

$f'(6) = (1)(11) > 0$

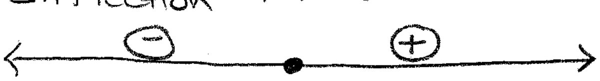
Rel Max at $x = -5$
Rel Min at $x = 5$
Increasing: $(-\infty, -5) \cup (5, \infty)$
Decreasing: $(-5, 5)$

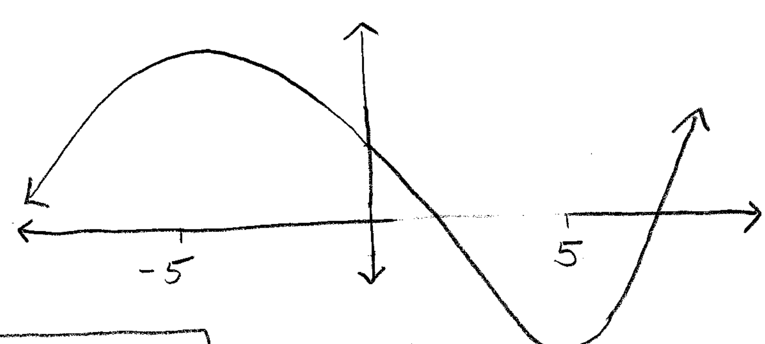
$f''(x) = 2x$

$0 = 2x$

$0 = x$

Inflection Pt at $x = 0$





Concave Down: $x < 0$, Concave Up: $x > 0$

2. You are operating a hotel, and from past data you have collected you know that the number of rooms you rent, n, is related to the price you charge per room p, by the following relationship:

$$n = 150 - p$$

What price should you charge per room to make your revenue as large as possible? Assume that p is between 0 and 150. <u>Hint:</u> revenue = number of rooms rented × price charged.

R = revenue

n = # rooms rented

p = price charged per room

$R = np \longleftarrow$ Objective Eq

$n = 150 - p \longleftarrow$ Constraint Eq

$R = (150 - p)p = 150p - p^2$

$R' = 150 - 2p$

$0 = 150 - 2p$

$2p = 150$

$p = 75$

$p = 75$ is a critical point of R. To make sure it's a maximum, find

$R(p=0)$, $R(p=75)$, $R(p=150)$

bounds given in problem

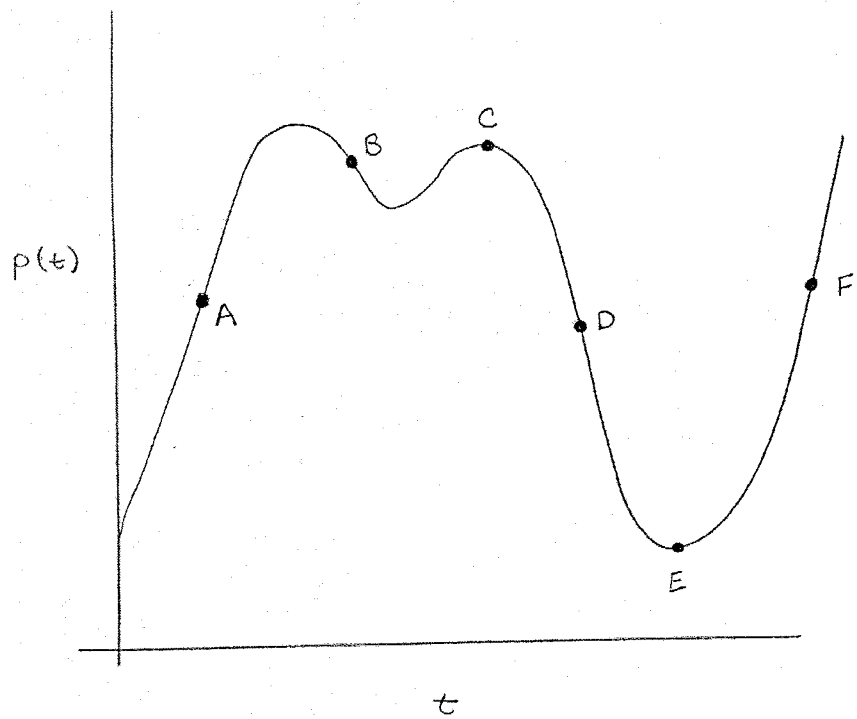
$R(p=0) = 0$

$R(p=75) = (150-75)75 = 75^2$

$R(p=150) = 0$

$\boxed{\text{Revenue is maximized by charging \$75/night}}$

3. A car is moving along a straight road. The position of the car, $p(t)$, is graphed below (t = time). Answer questions (a) through (e) about the movement of the car: (Hint: velocity is the derivative of position, acceleration is the 2nd derivative of position. Velocity means how fast the car is moving.)



To receive full credit, give explanations for your answers.

(a) Is the car moving faster at point A or point C?

Faster at A, since slope of tangent line bigger there

(b) How fast is the car moving at point E?

Not moving. Tangent line horizontal (0 slope)

(c) Is the acceleration positive or negative at point C?

Negative. Concave down at C so $a = p'' < 0$

(d) What direction is the car moving at point D?

Backwards. Tangent line has negative slope

(e) Is the velocity increasing or decreasing at point E?

Increasing. $v' = p'' > 0$ at E since graph is concave up at E

4. In this problem, consider some function $f(x)$. Be as precise as possible when answering the following questions.

(a) If you were to explain to someone in plain English what is meant by the derivative of $f(x)$, what would you tell them?

The derivative of $f(x)$ is the rate of change of $f(x)$

(b) Give an explanation of what $f'(x)$ is using tangent lines.

$f'(x)$ is the slope of the tangent line to $f(x)$ at the point $(x, f(x))$

(c) Give the official, mathematical definition of the derivative of $f(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

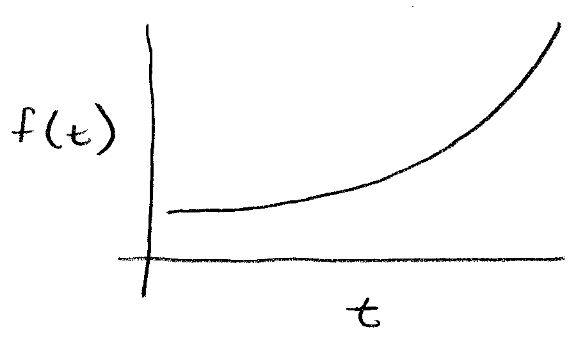
5. From 1940 to 1990, the number of parking tickets given out in Washington DC increased every year, and each year's increase was larger than the previous year's increase.

(a) If $f(t)$ is the number of parking tickets given out in Washington D.C. in year t, where t is between 1940 and 1990. Is $f'(t)$ positive or negative? Is $f''(t)$ positive or negative?

- $f'(t)$ positive since # of tickets/year is increasing
- $f''(t)$ positive since rate of increase is increasing

(b) Draw a rough graph of $f(t)$. Your graph will be graded on two things: (1) The 1st derivative of the curve you draw must have the correct sign and (2) The 2nd derivative of the curve you draw must have the correct sign.

Graph should be increasing and concave up



6. You are building a rectangular fence. One side of the fence will be facing the road, and city laws require you to use a fencing material that costs \$9 per linear foot. For the other three sides, you can use a cheaper fencing material that costs only \$3 per linear foot. If you only have \$240 total to build the fence, how should you construct the fence in a way to make the area inside the fence as large as possible?

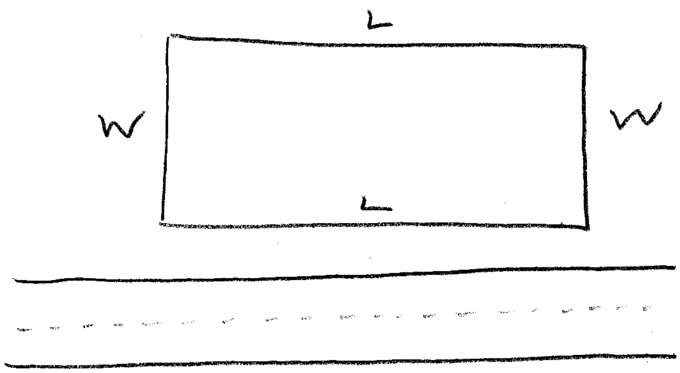
If you are having trouble with this problem, you may find it helpful to break things down into smaller steps:

1. Draw a picture and assign variable names to all quantities.
2. Write the objective equation.
3. Write the constraint equation.
4. Use the constraint equation to turn the objective equation into a function of one variable.
5. Find the maximum of the objective equation. Make sure you justify WHY it is the point you have found is a maximum of the objective equation.

(1) A = area inside fence

L = length of rectangle

W = width of rectangle



(2) Objective Eq: $A = LW$

(3) Constraint: $240 = 9L + 3W + 3L + 3W$

$240 = 12L + 6W$

(4) $6W = 240 - 12L$

$W = 40 - 2L$

$A = L(40 - 2L)$

$A = 40L - 2L^2$

(5) $0 = A' = 40 - 4L$

$4L = 40$

$L = 10$

$\boxed{W = 40 - 2L = 40 - 20 = 20 \quad \text{Area maximized by setting } L = 10,\ W = 20}$

To make sure this is a maximum, find $A(L=0)$, $A(L=10)$, $A(L=20)$.

largest possible length

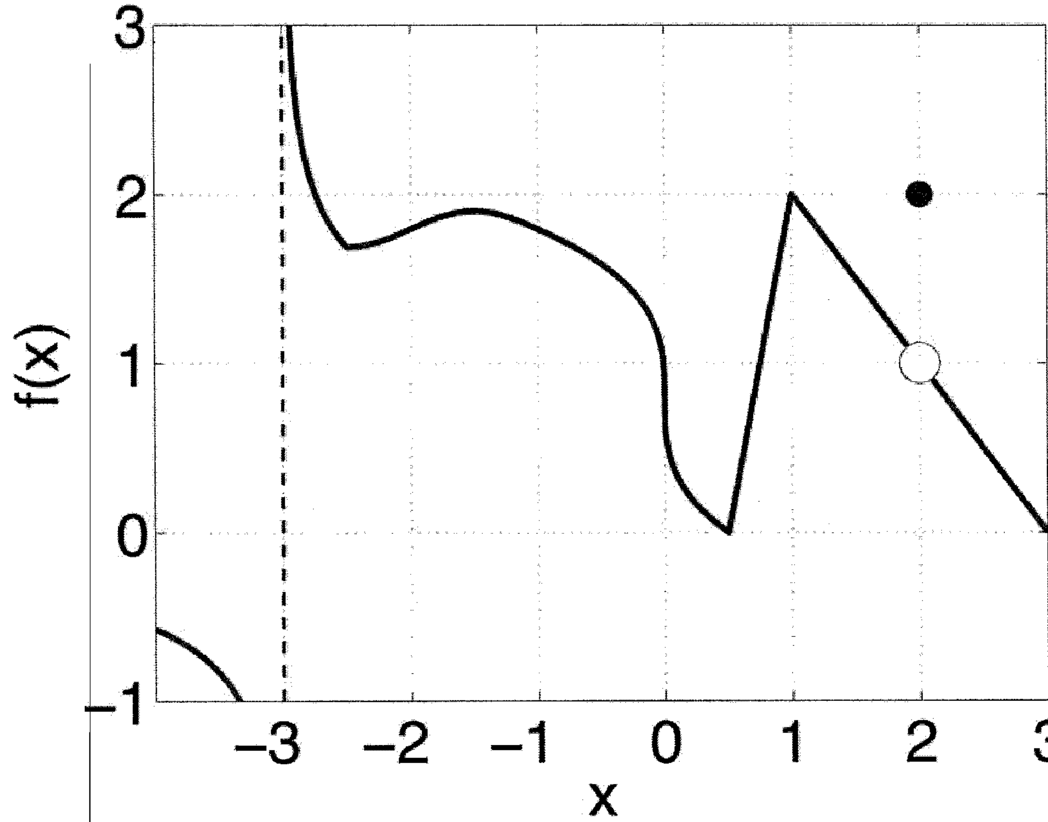
Smallest possible length

$A(L=0) = 0$

$A(L=10) = 400 - 200 = 200$ ✓ max at $L=10$

$A(L=20) = 0$

7. Consider the graph below



For each of the following four points (part (a) through part (d)), tell me (1) is $f(x)$ differentiable there? (2) Is $f(x)$ continuous there? If your answer is no to either of these questions, explain why.

(a) $x = -3$

Not continuous since $\lim_{x \to -3} f(x)$ DNE

Cant be differentiable since not continuous

(b) $x = -1$

Continuous and differentiable

(c) $x = 2$

Not continuous. $\lim_{x \to 1} f(x) = 1$, $f(1) = 2$

(d) $x = 0$

Continuous, but not differentiable due to vertical tangent lines

Honor Pledge: I have neither given nor received unauthorized aid on this test.

Signature: ______________________________

Bonus (5 pts): Find the derivative of the following. Be sure to show all of your work.

$$y = \frac{2x + (x + \sqrt{x^2+1})^{3/2}}{(x+2)(x+7)}$$

$$y = \frac{f(x)}{g(x)} \longrightarrow f(x) = 2x + (x + \sqrt{x^2+1})^{3/2}$$

$$\longrightarrow g(x) = (x+2)(x+7) = x^2 + 9x + 14$$

Quotient Rule

$$y' = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$f'(x) = 2 + \frac{3}{2}(x + \sqrt{x^2+1})^{1/2}\left(1 + \frac{1}{2}(x^2+1)^{-1/2}(2x)\right)$$

$$g'(x) = 2x + 9$$

$$\boxed{y' = \frac{\left\{(x^2+9x+14)\left[2 + \frac{3}{2}(x+\sqrt{x^2+1})^{1/2}(1 + x(x^2+1)^{-1/2})\right] - (2x + (x + \sqrt{x^2+1})^{3/2})(2x+9)\right\}}{[x^2+9x+14]^2}}$$