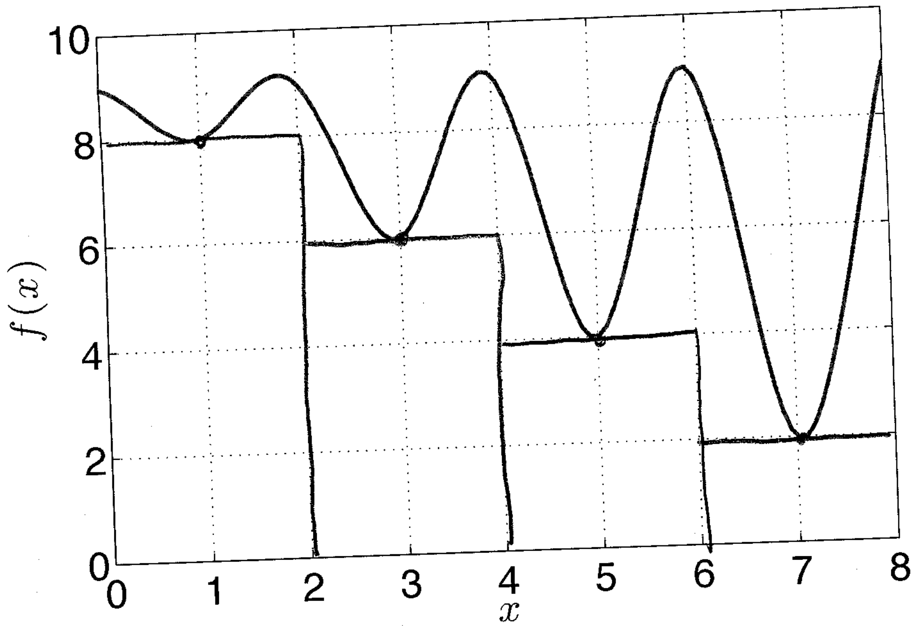
Name: Answer Key

MA 131 Test 4 Form B

1.) Below is the graph of some function $f(x)$.



a.) Approximate $\int_0^8 f(x)\,dx$ with a Riemann sum. Break the interval $[0,8]$ into 4 subintervals, and use the function value of $f(x)$ at the *midpoint* of each interval to determine the height of each rectangle. **[10 points]**

$$\int_0^8 f(x)\,dx \approx 2\cdot 8 + 2\cdot 6 + 2\cdot 4 + 2\cdot 2$$

$$= 16 + 12 + 8 + 4$$

$$= 28 + 12$$

$$= \boxed{40}$$

b.) Is the approximation you obtained in part (a): larger than $\int_0^8 f(x)\,dx$, smaller than $\int_0^8 f(x)\,dx$, or exactly equal to $\int_0^8 f(x)\,dx$. Why? **[4 points]**

Smaller. Area of rectangles is less than area under the curve $y = f(x)$

2. a) Which of the following is the *antiderivative* of $f(x) = \ln(x)$? Why? **[10 points]**

(a) $\frac{1}{x} + C$ (b) $x \cdot \ln(x) - x + C$ (c) $\frac{1}{2} \cdot (\ln(x))^2 + C$

$\frac{d}{dx}\left(\frac{1}{x} + c\right) = \frac{d}{dx} x^{-1} = -x^{-2} = \frac{-1}{x^2} \neq \ln x$ NOT (a)

$\frac{d}{dx}(x\ln x - x + c) = x\left(\frac{1}{x}\right) + (1)\ln x - 1 = 1 + \ln x - 1 = \ln x$ ✓

$\frac{d}{dx}\left[\frac{1}{2}(\ln x)^2 + c\right] = \frac{2}{2}\ln x \frac{d}{dx}\ln x = \frac{\ln x}{x}$ NOT (c)

$\boxed{x\ln x - x + C}$ is antiderivative of $\ln x$

since $\frac{d}{dx}(x\ln x - x + c) = \ln x$

b.) Find the following definite integral. Evaluate all natural logs and write your final answer as a single whole number. **[8 points]**

$$\int_1^e \ln(x)\, dx$$

Use Fundamental Theorem of Calculus

$$\int_1^e \ln x\, dx = x\ln x - x \Big|_1^e$$

$$= \underbrace{e\ln e}_{=1} - e - (1\underbrace{\ln 1}_{=0} - 1)$$

$$= e - e - (0 - 1)$$

$$= \boxed{1}$$

3.) Find the volume of the solid obtained by revolving $f(x) = x\sqrt{x^3+2}$ around the x-axis from $x = 0$ to $x = 1$. **[16 points] Bonus: [1 point]** Sketch this solid.

$$V = \pi \int_0^1 x^2(x^3+2)\,dx$$

$u = x^3 + 2$

$du = 3x^2 dx$

$\frac{1}{3} du = x^2 dx$

$x = 0, \quad u = 2$

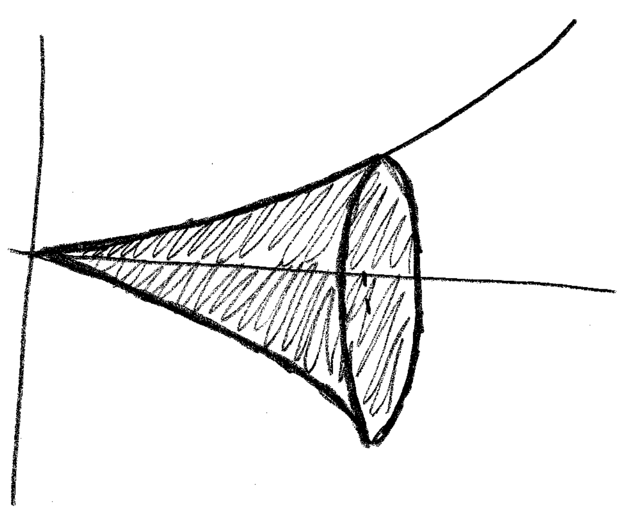
$x = 1, \quad u = 3$

$$= \frac{\pi}{3} \int_2^3 u\,du$$

$$= \frac{\pi}{3} \frac{u^2}{2} \Big|_2^3$$

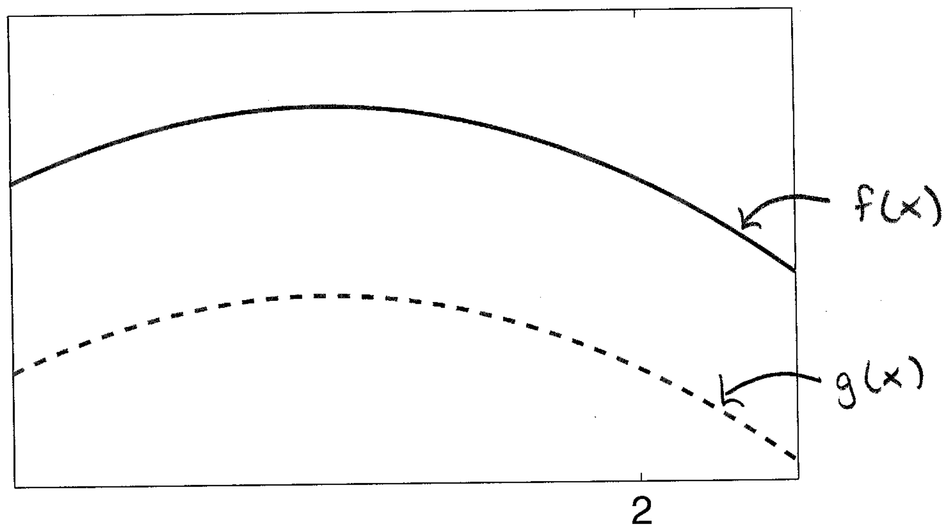
$$= \frac{\pi}{3}\left(\frac{9}{2} - \frac{4}{2}\right)$$

$$= \frac{\pi}{3}\left(\frac{5}{2}\right) = \boxed{\pi \frac{5}{6}}$$



x	$f(x)$
0	0
1	$\sqrt{3}$
2	$2 \cdot \sqrt{10}$

4.) In the graph below, the function $g(x)$ was obtained by shifting the graph of $f(x)$ down by 1. In the graph below, $g(x)$ is the dashed line, and $f(x)$ is the solid line



If $f'(2) = -1$, what is $g'(2)$? Why? [**8 points**]

$$g(x) = f(x) - 1$$

$$g'(x) = f'(x)$$

$$\boxed{g'(2) = f'(2) = -1}$$

Shifting f down by 1 leaves its derivative <u>unchanged</u>

5.) A rocket is launched from the ground, and its velocity as a function of time $v(t) = -t^2 + 16$ (t is in seconds, $v(t)$ is in ft/sec). If $v(t)$ is positive, then this indicates that the rocket is moving upwards, and a negative velocity indicates that the rocket is moving down towards the ground.

a.) When is the rocket moving upwards? Downwards? **[6 points]**

$0 < -t^2 + 16$

$t^2 < 16$

$t < 4$

Upwards when $t < 4$

$0 > -t^2 + 16$

$t^2 > 16$

$t > 4$

Downwords when $t > 4$

b.) At what value of t does the rocket reach its peak height? **[4 points]**

$\boxed{t = 4}$

c.) Compute the area under the velocity curve from time $t = 0$ to time $t = t^*$, where t^* is the answer you found for part (b). For example, if your answer to part (b) was 2, find the area under the velocity curve from time $t = 0$ to time $t = 2$. **[10 points]**

$$\int_0^4 (-t^2 + 16)\,dt = -\frac{t^3}{3} + 16t \Big|_0^4$$

$$= -\frac{64}{3} + 64$$

$$= \frac{-64}{3} + \frac{192}{3}$$

$$= \boxed{\frac{128}{3}}$$

d.) What does your answer to part (c) represent about the rocket? DO NOT just say it's the area under the velocity curve. **[6 points]**

The maximum height of the rocket is $\frac{128}{3}$ ft

6.) Water is flowing into a tank at a rate of $r(t)$ gallons per hour, where $r(t) = 10 - t$.

a.) How much water flows into the tank from time $t = 0$ to time $t = 4$? **[10 points]**

$$\int_0^4 (10-t)\,dt = 10t - \frac{t^2}{2}\Big|_0^4$$

$$= 10(4) - \frac{16}{2} - \left(10\cdot 0 - \frac{0^2}{2}\right)$$

$$= 40 - 8$$

$$= \boxed{32 \text{ gallons}}$$

b.) What is the *average* rate at which the water flows into the tank from $t = 0$ to $t = 4$? **[8 points]**

$$\frac{1}{4-0}\int_0^4 r(t)\,dt = \frac{1}{4}\int_0^4 (10-t)\,dt$$

$$= \frac{1}{4}(32)$$

$$= \boxed{8 \ \frac{\text{gallons}}{\text{hour}}}$$