3 Definitions of the Derivative

1. (*English*) - Consider some function $f(x)$. The derivative, written $f'(x)$, is the rate of change of $f(x)$.
2. (*Tangent Lines*) - The derivative at a point is equal to the slope of the tangent line at that point
3. (*Limit*)

Each of these 3 definitions means exactly the same thing, but are all useful for different reasons.

1. **English Definition** - this is a real world description of what the derivative means
2. **Tangent Line Definition** - estimate derivative by looking at graphs (Quiz 1, Problem 3)
3. **Limit Definition** - As we'll see below, this definition allows us to actually compute the derivative (English and Tangent Line Definitions don't let us do this)

Example: Let $f(x) = x^2$. Find $f'(x)$

Two ways to write the derivative of $f(x)$ (they mean exactly the same thing)

1.
2.

Example: Find $\frac{d}{dx}x^3$. Hint: $(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$

$f(x)$	1	x	x^2	x^3
$f'(x)$				

Power Rule: Let n be any number and $f(x) = x^n$. Then $f'(x) = nx^{n-1}$

Example: Let $f(x) = x^{11}, g(x) = x^{-4}, h(x) = x^{1/2}$. Find $f'(x), g'(x)$, and $h'(x)$.

Example: Let $f(x) = \sqrt[3]{x}, g(x) = \dfrac{1}{x^2}$. Find $f'(x), g'(x)$.

Constant Multiple Rule: Let k be any constant

$$\frac{d}{dx}\left[k \cdot f(x)\right] = k \cdot \frac{d}{dx}[f(x)]$$

Example: Let $f(x) = 7x^2, g(x) = 3x^{-4}$. Find $f'(x)$ and $g'(x)$.

Sum Rule:

$$\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

Example: Let $f(x) = x^2 + x + 1, g(x) = 3x + \frac{2}{x}$. Find $f'(x)$ and $g'(x)$.

General Power Rule:

$$\frac{d}{dx}\left([g(x)]^n\right) = n \cdot [g(x)]^{n-1}\frac{d}{dx}g(x)$$

Example: Let $f(x) = (x+1)^{17}, g(x) = \sqrt{x^2+2x}$. Find $f'(x)$ and $g'(x)$.

WHAT YOU SHOULD KNOW

1. All 3 definitions of the derivative
2. If I give you a function, you need to be able to compute the derivative

Practice Problems

Find the derivatives of the following functions:

1. x^5
2. x^{32}
3. x^2
4. x^7
5. $2x + 1$
6. $4x^3$
7. $7x + 3$
8. $5x^2 + 8x + 1$
9. $2x^5 + x^7$

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1. $x^{3/2}$
2. x^{-3}
3. x^{-6}
4. $3x^{-2} + 2x^{1/2}$
5. $5x^{1/2} + 2x^{-3}$
6. $\dfrac{1}{x^2}$
7. $7\sqrt{x}$
8. $\dfrac{2}{x} + 2x + 1 + 3\sqrt{x}$
9. $\dfrac{1}{\sqrt{x}} + \dfrac{4\pi}{x^3} + 3x^2$

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1. $\sqrt{x^2 + 4x}$
2. $(x^2 + 1)^7$
3. $\dfrac{1}{x + x^3}$
4. $(\sqrt{x} + x)^3$
5. $\sqrt{5x + 7}$
6. $\sqrt{x^2 + x + 1}$
7. $\dfrac{1}{2x^2 + 119}$
8. $(3x^4 + 11x)^5$
9. $\dfrac{1}{4\sqrt{x} + x^2}$

Extra Credit Problems

1. 1
2. $4x^3 + 2x^2 + 7$
3. $5\sqrt{x}$
4. $\dfrac{1}{x^3}$
5. x^8
6. $\sqrt{x^2 + x + 1}$
7. $3x^3 + 9$
8. $\dfrac{5}{x^2} + 2\sqrt{x}$
9. $\dfrac{1}{4x + x^2}$