Announcements

- HW #1 Due Thursday, Jan 17
  - Section 1.1 #1, 6, 8, 12, 31, 42, 47 (See Sec 1.1, Exercise 1), 59, 60
  - Section 1.2 #2, 4, 5, 6, 7, 8, 9, 10, 14, 37, 38
- Quiz #1 In Class Thursday, Jan 17
- HW #2 Due Tuesday, Jan 22
- Course Webpage: www.willcousins.com then click on MA131 link

---

Point Slope Form of Line

Let's say you know that the slope of a line is m, and that (x_1, y_1) is a point on that line. How can you find the equation of this line?

Point-Slope Form of a Line

$$y - y_1 = m(x - x_1)$$

Example #1: Write the equation of a line with slope 4 where the point (5,92) is on the line. What is the y-intercept of this line?

Example #2: Write the equation of the line passing through (1,3) and (5,11).

We know how to compute the slope from last time:

We can use the point-slope form to write the equation of this line:

Rates of Change

From last time,

Definition: The <u>average rate of change</u> of a function $f(x)$ from x_1 to x_2 is

Remarks:

- For lines ($f(x) = mx + b$), the average rate of change from x_1 to x_2 is the slope, m. It does not matter what values you pick for x_1 and x_2.
- If $f(x)$ is not a line, the average rate of change depends on what you pick for x_1 and x_2

Example #3: Average rate of change for a line

Let t denote time (in year) and $h(t)$ be Mark's height in inches at time t. Suppose Mark grows 3 inches per year and he is 20 inches tall when he is born. This means $h(t)$ and t are related by the following line:

On average, how fast is Mark growing from ages 1 to 4?

How about from ages 5 to 7?

Example #4: Average rate of change for nonlinear function

Let $f(x) = x^2$. What is the average rate of change of f from $x = 1$ to $x = 4$?

What's the average rate of change from $x = 5$ to $x = 7$?

Example #5: Meaning of rates of change

Jimmy works as a knife salesman, and his weekly salary in dollars (s) is related to the number of knives he sells (k) by the following line:

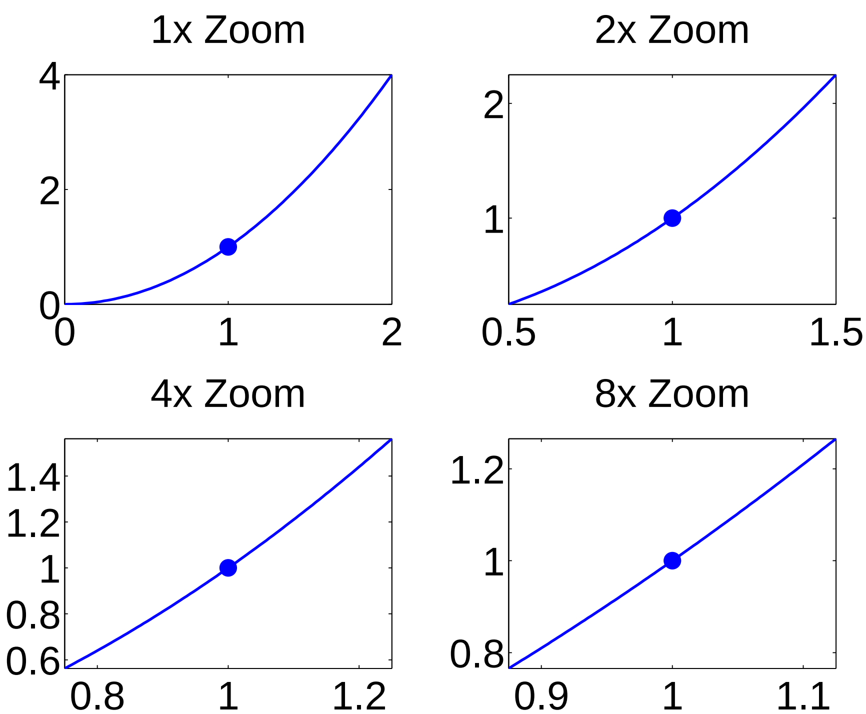
$$s = 10k + 100$$

This week Jimmy sold 40 knives. How much money did he make this week?

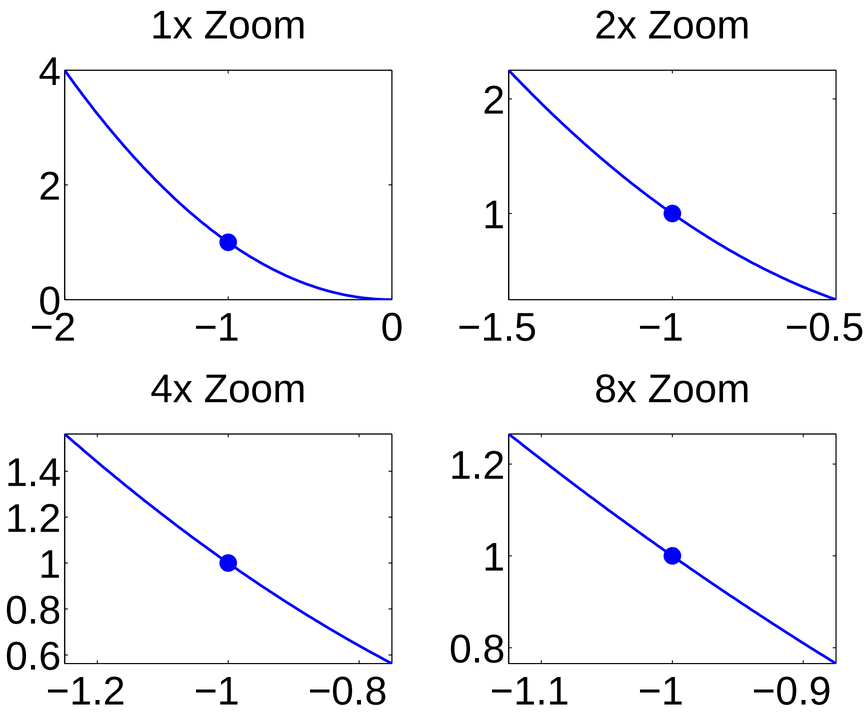
Jimmy thinks he can work a little harder next week and sell 10 more knives. How much will this change Jimmy's salary for next week?

Jimmy thinks that this isn't enough money to pay his bills, and he decides he needs to increase his salary by 150 next week. How many more knives will he need to sell to do this?

Lines are easy–we have one number that tells us the rate of change (the slope). Things are more complicated for functions that aren't lines: the average rate of change can be different depending on which values of x we care about. However, we can make sense of things if we only consider one point at at time. Let's zoom in on the graph of $f(x) = x^2$ at the point $(1, 1)$.



Now let's zoom in on (-1,1).



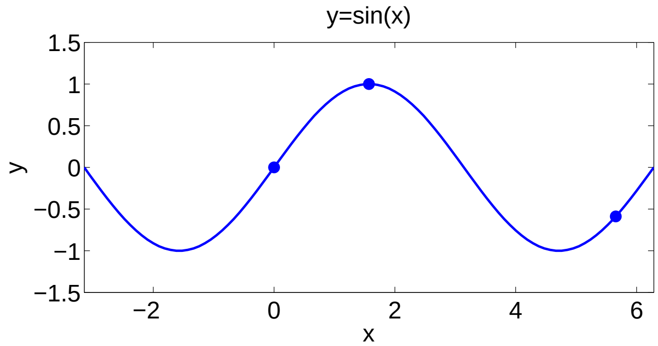
Tangent Lines

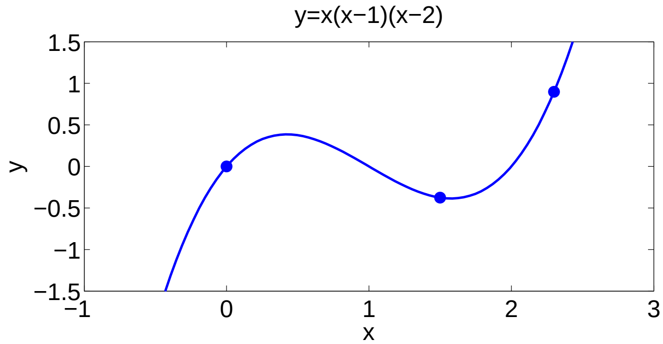
The line we start to see when we zoom in enough is called the tangent line.

How to draw the **tangent line to $f(x)$ at x**

- The tangent line must intersect the graph of $f(x)$ at $(x, f(x))$
- The slope of the tangent line should be the slope of the line that appears when you closely zoom in on the point $(x, f(x))$

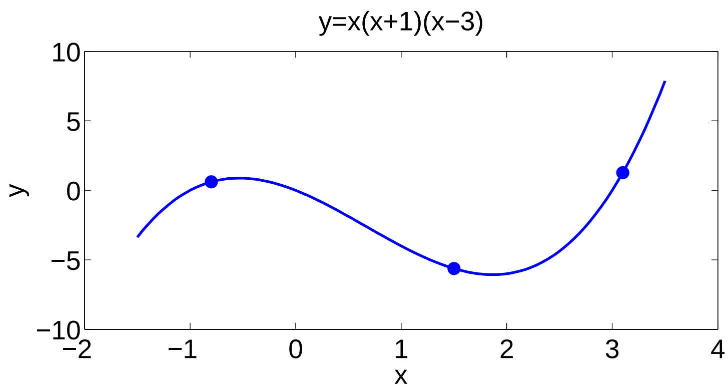
Example #6: Draw tangent lines at the highlighted points for the following two graphs

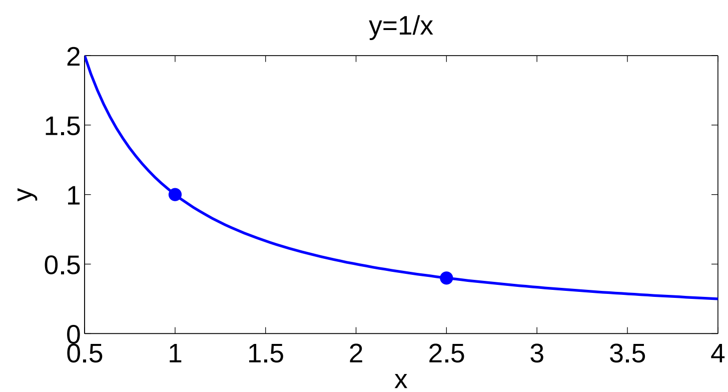




Good reference tool: `www4.ncsu.edu/~lmvanbla/index.html` (Go to Teaching(top right), Interactive Examples, then scroll to bottom of page until you see "Relating Tangent Lines and Derivatives"). Thanks to Luke Van Blaircum.

Example #7: Draw tangent lines at the highlighted points for the following two graphs





Definition: The rate of change of a curve at a point is the slope of a tangent line at that point. In math terms, this says that the rate of change of $f(x)$ at x is the slope of the tangent line to f at x.

By now you should understand how to draw the tangent line to a function at a point. You should be able to tell if the tangent line has a positive or negative slope, but you won't be able to determine the exact value of the slope of the tangent line just by looking at the graph. To figure out the precise value of the rate of change of a function at a point, we will need to understand secant lines.

Secant Lines

Definition: Let $(a, f(a))$ and $(b, f(b))$ be two points on the graph of f. The secant line is the the line connecting these two points.

Let's draw the secant line connecting $(x, f(x))$ and a nearby point:

What happens if we let h get smaller and smaller?

`www4.ncsu.edu/~lmvanbla/index.html` has another nice tool for visualizing this. Use the "Relating Secant Line to Tangent Line" applet.

Relationship between secant and tangent lines: As h gets smaller and smaller, the secant line through $(x, f(x))$ and $(x + h, f(x + h))$ gets closer and closer to the tangent line to f at x.

What's the formula for the slope of the secant line through $(x, f(x))$ and $(x + h, f(x + h))$?

Example #8: Let $f(x) = x^2$. What is the rate of change of f when $x = 1, x = 2$, and $x = 4$?

Example #9: Suppose the population of rabbits (p) can be expressed as the following function of time (t), where the units for t are months:

$$p(t) = 2^t$$

When $t = 1$, how fast is the population of rabbits increasing? What about when $t = 3$? $t = 5$?

Summary

Things you should know:

- Average rate of change: know definition and be able to compute it
- Tangent lines: be able to draw them
- Secant lines
  - Know definition and how to compute slope
  - Know connection to tangent lines
- Rate of change at a point
  - Know definition
  - Be able to compute by computing slope of secant line through $(x, f(x))$ and $(x+h, f(x+h))$ and letting h become small