Optimization

Commonly a mathematical function $f(x)$ has the following real world meaning:

- x = something you control
- $f(x)$ = something you really care about

Examples

- $P(s)$, where $P(s)$ is the profit an airline makes, and s is the speed their aircraft travel
- $T(r)$, where $T(r)$ is tax revenue and r = tax rate

Optimization

If $f(x)$ is something *good* (Ex: profits, happiness), what value of x should I pick to make $f(x)$ as *large* as possible?

If $f(x)$ is something *bad* (Ex: carbon emissions, size of brain tumor), what value of x should I pick to make $f(x)$ as *small* as possible?

Example 1: You are building a fenced in, rectangular garden, and you have 40 ft of fence you can use. How can you set up your garden so that the space inside the fence is as big as possible?

Steps for solving an optimization problem

1. Draw a picture (if possible)
2. Decide what quantity Q is to be maximized or minimized
3. Assign letters to all quantities
4. Determine the objective equation that expresses Q as a function of all other variables
5. Determine the constraint equation that relates the variables in the right hand side of the objective equation
6. Use the constraint equation to simplify the objective equation to an equation of one variable.
7. Determine any bounds and find the absolute maximum or minimum of the equation in step 6

Example 2: Suppose a TV manufacturer knows that if they charge p dollars for a TV, they will sell n = 10,000-10p TV's per month. What is the largest amount of monthly revenue they can make? How much should they charge for a TV, and how many should they make?

Practice Problems

1. Suppose that in Example 2, the cost to produce n TVs in a month is $1000 + 100n$. How many TVs should the company produce to maximize their *profit*? How much profit will they make?

2. Suppose you are running a hotel, and you know that if your charge p dollars per night, you will book $n = 200 - 1.5p$ rooms. What price should you charge to make your revenue as large as possible? Assume you have 100 rooms total in your hotel.

3. A homeowner wishes to add a room of 120 square feet to his house. The cost of building the exterior wall is $80 per linear foot, and the cost of removing the interior wall is $50 per linear foot. What are the optimal dimensions?

4. An aluminum manufacturing company is working on designs for soup cans. In one design they are going to use the same gauge metal for the whole can, it costs $0.002 per square centimeter of aluminum. If the can must hold 500ml of soup what dimensions will minimize the cost of the can.

5. A walnut grower estimates from past records that if 20 trees are planted, each tree will yield 60 pounds of nuts per year. If for each additional tree planted the average yield per tree drops 2 pounds, how many trees should be planted to maximize the amount of walnuts produced? What is the maximum number of walnuts? Assume that the grower has room to plant up to 50 trees.

6. A candy box is to be made out of a rectangular piece of cardboard that measures 8 by 12 inches. Squares of equal size will be cut out of each corner, and then the ends and sides will be folded up to form a rectangular box. What size square should be cut from each corner to obtain a maximum volume?

7. Two heavy industrial areas are located 10 miles apart (one at $x = 0$, another at $x = 10$). Suppose the concentration of pollutant is related to x in the following way:

$$C(x) = \frac{8}{x^2} + \frac{1}{(10+x)^2}$$

Suppose a school is being built somewhere between these two factories, and you are in charge of deciding where to build it. Where would you decide to build the school?

8. Suppose you are running a farm, and you would like to build 2 rectangular fenced in areas of the same size: one for your pigs, and one for your sheep. If you have 1000 feet of fence to work with, how should you construct your fence so the sheep and pigs have as much room as possible?